ATTITUDE CONTROL OF ARTICULATED, FLEXIBLE SPACECRAFT

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Summary: This paper extends the authors' prior work on the regulation of flexible space structures via partial feedback linearization methods to articulated systems. The modeling of such systems through Poincaré's form of Lagrange's equations is shown to be efficient and to provide for easy construction of feedback linearizing control laws. An application to a flexible space platform with a mobile remote manipulator is included.

Keywords: flexible spacecraft, Poincaré's equations, feedback linearization, adaptive control

1 Introduction

Modern spacecraft typically consists of several articulating bodies some of which have considerable flexibility. One such system is the Space Station Freedom with a Mobile Remote Manipulator System (SSF/MRMS). The issue of attitude control for this configuration has received attention in the literature, e.g., the papers of Mah and his coworkers [9] and Wie et al [14]. The problem of primary interest is the attitude regulation of the space station while the MRMS undergoes arbitrary prescribed maneuvers. In [1,14], the main issues addressed have to do with the ability of the attitude regulator to reject long term periodic disturbances due to environmental torque including gravity gradient torque and cyclic aerodynamic torques. Both of these studies outline the potential benefits of LQG design for this controller. They also show the sensitivity of attitude control performance to MRMS motion. In fact, Wie et al show that large MRMS motion can destabilize the attitude control system because of the consequent changes in the system inertia and suggest the need for gain scheduling of linear controllers. The essential interactions are characterized by highly nonlinear dynamics that appear to be ideally suited for application of feedback linearization methods.

Our goal is to describe recently developed innovations for modeling and control of articulated systems and to demonstrate their application to a spacecraft configuration representative of the SSF/MRMS. The attitude control issues addressed herein are related to those defined in [9,13] except that we focus on the short time scale problem (time scale of minutes) associated with MRMS motion whereas in the aforementioned works MRMS induced disturbances are considered but primarily for their affect on long term behavior (time scale of orbits). We consider control system design for decoupling and stabilization with respect to MRMS motion. Since the control problem of interest heren evolves on a short time scale, we do not include environment (orbital frequency) disturbance torques in our analysis.

The methods considered herein address the essential nonlinearity of these systems directly. A unified approach to modeling and nonlinear control system design is employed. This paper extends the authors' prior work [2,3] to articulated systems. In particular, this paper complements [2] in which we discuss nonarticulating, multibody, flexible systems. In Sections 2&3, we describe our approach to modeling articulated, flexible structures via Poincaré's equations. In Section 4 we provide feedback linearizing attitude control laws for this class of models and in Section 5 simulation results are summarized for a prototype SSF/MRMS.

2 Lagrange's Equations & Quasi-Coordinates

Our approach to multi-flex-body modeling is based on a Lagrangian framework in which the Lagrangian dynamics are conveniently formulated using quasi-coordinates [1,4,10] leading to a system of equations often called Poincaré's equations. The method has been further developed using finite element analysis [2] for reduction to finite dimensions and the recursive constructions introduced by Rodriguez and Jain [7,12] for serial chains of articulating bodies. The resultant equations are suitable for analysis, computation and control system design.

2.1 The Euler-Lagrange Equations

The formalism of Lagrangian dynamics begins with the identification of the configuration space, i.e. the generalized coordinates, associated with the dynamical system of interest. Once the configuration manifold, M, is specified we have the natural definition of velocity at a point $q \in M$ as an element, $\dot{q}$, in the tangent space to M at q, often denoted T_qM. We then define the state space as the union of tangent spaces at all points $q \in M$, the so-called tangent bundle TM. The evolution of the system in the state space is characterized by the definition of a Lagrangian $\mathcal{L}(q,\dot{q})$: $TM \rightarrow \mathbb{R}$ and use of Hamilton's principle of least action. The motion of a dynamical system subject to external generalized forces, Q, between times t_1 and t_2 is a "natural" motion if and only if

$$\int_{t_1}^{t_2} \left(\delta\mathcal{L} + Q^t \delta q \right) dt = 0 \qquad (2.1)$$

If the coordinate variations are independent, then we obtain the Euler-Lagrange equations

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = Q \qquad (2.2)$$

or, in the usual case where $\mathcal{L}(q,\dot{q}) = \mathcal{T}(q,\dot{q}) - \mathcal{V}(q)$, where $\mathcal{T}$ and $\mathcal{V}$ are the kinetic energy and potential energy functions, respectively, it may be convenient to write

$$\frac{d}{dt}\frac{\partial \mathcal{T}}{\partial \dot{q}} - \frac{\partial \mathcal{T}}{\partial q} + \frac{\partial \mathcal{V}}{\partial q} = Q \qquad (2.3)$$

2.2 Quasi-Coordinates & Alternate Equations of Motion

It is well known that in some cases it is easier to formulate the equations of motion in terms of velocity variables which can not be expressed as the time derivatives of any corresponding configuration coordinates. Such velocities are called *quasi-velocities* and are often associated with so-called *quasi-coordinates.* Quasi-velocities are meaningful physical quantities. The angular velocity of a rigid body is a prime example. Quasi-coordinates are not meaningful physical quantities. They make sense only in terms of infinitesimal motions. The notion of quasi-velocities and quasi-coordinates leads to a generalization of Lagrange's equations which is applicable to systems with nonholonomic as well as the usual holonomic constraints. Such generalizations were produced at the turn of the century, e.g. [1, 11].

Let M be the m-dimensional configuration manifold for a Lagrangian system and suppose $v_1,...,v_m$ constitute a system of m linearly independent vector fields on M. Then each commutator can be expressed

$$[v_i,v_j] = \sum_{k=1}^{m} c_{ij}^k(q) v_k \qquad (2.4)$$

Indeed, the coefficients are easily computed. Define

$$V := [v_1\ v_2 \dots v_m],\ U = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix} := V^{-1},\ \chi_{ij} = [c^1_{ij}\ c^2_{ij} \dots c^m_{ij}]^t \quad (2.5)$$

Then (2.4) yields

$$\chi_{ij} = U[v_i,v_j] \ \text{ or } \ c^k_{ij} = u_k[v_i,v_j] \quad (2.6)$$

Suppose $q(t)$: $[t_1,t_2] \rightarrow M$ is a smooth path, then $\dot q(t)$ denotes the tangent vector to the path at the point $q(t) \in M$. Thus, we can always express $\dot q$ as a linear combination of the tangent vectors v_i, $i=1,..,m$

$$\dot q = V(q)p \quad (2.7a)$$

where

$$p = U(q)\dot q \quad (2.7b)$$

The variables p are called *quasi-velocities.* We might try to associate these "velocities" with a set of coordinates π, in the sense that $\dot\pi = p$. This is not always possible because the right hand side of (2.7b) may not be an exact differential. Set $\mathcal{L}(q,p) = \mathcal{L}(q,\dot q)|_{\dot q = V(q)p}$. In terms of $\mathcal{L}$ Lagrange's equations are attainable in the form given by the following proposition.

Proposition 2.1: Hamilton's principles leads to the equations of motion in terms of the coordinates q, p

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial p} - \sum_{j=1}^{m} p_j \frac{\partial \mathcal{L}}{\partial p} U X_j - \frac{\partial \mathcal{L}}{\partial q} V = Q^t V \quad (2.8)$$

where $X_j = [[v_j,v_1]\ [v_j,v_2] \dots [v_j,v_m]]$.

proof: see Arnold et al [3].

Remarks:

(1) These equations are referred to as Poincare's equations [1,4]. They are related to Caplygin's equations in quasi-coordinates and to the Boltzman-Hamel equations [11], and also to the generalized Lagrange equations of Noble [8].

(2) Poincare's equations (2.8) along with (2.7a) form a closed system of first order differential equations which may be written in the form

$$\dot q = V(q)p \quad (2.9a)$$

$$\dot p \frac{\partial^2 \mathcal{L}}{\partial p^2} = -p^t V^t(q) \frac{\partial^2 \mathcal{L}}{\partial q^t \partial p} + \sum_{j=1}^{m} p_j \frac{\partial \mathcal{L}}{\partial p} U X_j + \frac{\partial \mathcal{L}}{\partial q} V + Q^t V \quad (2.9b)$$

3 Algorithmic Formulation of Kinetic Energy

Systematic methods for the formulation of the equations of motion for complex mechanical systems are now receiving considerable attention. Although there are important historical precedents, the investigations most relevant to us are those of Rodriguez and his coworkers [7,12], who have formulated certain recursive techniques for rigid body systems and also for systems with flexible elements. In the following paragraphs we summarize the necessary concepts and explain how they are integrated into the Lagrangian framework. The key issue is the formulation of the kinetic energy function and we focus on that construction.

We adopt the convention, by which any vector $a \in \mathbb{R}^3$ is converted into a skew-symmetric matrix $\tilde a(a)$:

$$\tilde a(a) := \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \quad (3.1)$$

just as commonly done for angular velocity. Rodriguez et al [7,12] define the *spatial velocity at point C* of any body-fixed reference frame with origin at point C as $V_c := [\omega, v_c]$ where v_c is the velocity of point C and ω is the angular velocity of the body. Let O be another point in the same body and let r_{co} denote the location of C in the body frame with origin at O. Then the spatial velocity at point C is related to that at O by the relation

$$V_c = \phi(r_{co})V_o \quad (3.2)$$

where

$$\phi(r_{co}) := \begin{bmatrix} I & 0 \\ -\tilde r_{co} & I \end{bmatrix}, \text{ and its adjoint } \phi^*(r_{co}) := \begin{bmatrix} I & \tilde r_{co} \\ 0 & I \end{bmatrix} \quad (3.3)$$

Consider a serial chain composed of K+1 rigid bodies connected by joints as illustrated in Figure 1. The bodies are numbered 0 through K, with 0 denoting the base or reference body, which may represent any convenient inertial reference frame. The k^{th} joint connects body k-1 at the point C_{k-1} with body k at the point O_k.

Let a reference frame $\mathcal{F}^k$, with origin at O_k, be so oriented that its z-axis passes through C_k in the undeformed configuration. We will use a coordinate specific notation in which vectors represented in f^i (or its tangent space) will be identified with a superscript "i". Coordinate free relations carry no superscript. We assume that each link is a one dimensional, beam-like, flexible body and that the deformable centerline is coincident with the z-axis of $\mathcal{F}^k$ in the undeformed configuration. The beam equations will be written in the frame $\mathcal{F}^k$. We attach $\mathcal{F}^k$ to the body by requiring cantilever beam boundary conditions at z=0. Thus, $\mathcal{F}^k$ may be thought of as fixed in an infinitesimal element at O_k. We assume that deformations are small. Let F^k be a second reference frame with origin at C_k and aligned with $\mathcal{F}^k$ in the undeformed configuration. As located in $\mathcal{F}^k$ the point C_k has coordinates x=0, y=0, at $z=z_c$ in the undeformed configuration. The orientation of F^k under deformation is defined by fixing F^k in an infinitesimal element at C_k, i.e., the location of its origin in $\mathcal{F}^k$ is $\eta^k(z_c)$ and its relative angular orientation with respect to $\mathcal{F}^k$ is $\xi^k(z_c)$.

The k^{th} joint has n_k, $1 \le n_k \le 6$ degrees of freedom which can be characterized by n_k quasi-velocities $\beta(k)$ and a joint map matrix $H(k) \in \mathbb{R}^{6 \times n_k}$ so that $V_{O_k} - V_{C_{k-1}} = H(k)\beta(k)$. Let the *joint configuration parameters* be denoted by $\sigma_k \in \mathbb{R}^{n_k}$. The joint rotation matrix which defines the relative orientation of $\mathcal{F}^k$ with respect to F^{k-1} can be realized as a function of σ_k, which we denote $L(\sigma_k)$. The configuration rates $\dot\sigma_k$ are related to the quasi-velocities by a relation

$$\dot\sigma_k = \Sigma_k(\sigma_k)\beta(k) \quad (3.4)$$

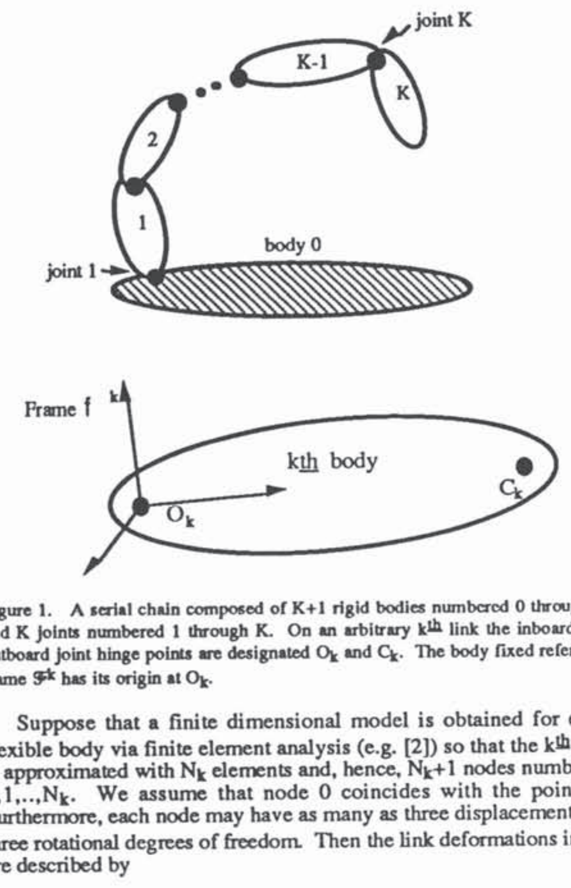

Figure 1. A serial chain composed of K+1 rigid bodies numbered 0 through K and K joints numbered 1 through K. On an arbitrary k^{th} link the inboard and outboard joint hinge points are designated O_k and C_k. The body fixed reference frame $\mathcal{F}^k$ has its origin at O_k.

Suppose that a finite dimensional model is obtained for each flexible body via finite element analysis (e.g. [2]) so that the k^{th} link is approximated with N_k elements and, hence, N_k+1 nodes numbered $0,1,..,N_k$. We assume that node 0 coincides with the point O. Furthermore, each node may have as many as three displacement and three rotational degrees of freedom. Then the link deformations in $\mathcal{F}^k$ are described by

$$\begin{bmatrix} \xi(z,t) \\ \eta(z,t) \end{bmatrix} = \begin{bmatrix} \hat\xi(t) \\ \hat\eta(t) \end{bmatrix} \Psi(z) \quad (3.5)$$

The relative (that is, with respect to the frame $\mathcal{F}^k$) spatial velocity associated with an infinitesimal element located at an arbitrary z in the undeformed configuration is

$$\upsilon(k,z) := \begin{bmatrix} \dot\xi(z,t) \\ \dot\eta(z,t) \end{bmatrix} = \begin{bmatrix} \dot{\hat\xi}(t) \\ \dot{\hat\eta}(t) \end{bmatrix} \Psi(z) = [\upsilon_1(k), \dots \upsilon_{N_k}(k)]\Psi(z) \quad (3.6)$$

where the columns of $[\upsilon_1(k),..\upsilon_{N_k}(k)]$ are the nodal spatial velocities, in particular, at node i ($z = z_i$) we use the notation

$$\upsilon_i(k) := \begin{bmatrix} \xi(z_i,t) \\ \dot{\eta}(z_i,t) \end{bmatrix} = \begin{bmatrix} \dot{\xi}_i \\ \dot{\eta}_i \end{bmatrix}, \quad i = 1,..N_k, \quad \upsilon_0(k) = 0 \qquad (3.7)$$

It is convenient to again use stacked notation and define $\upsilon(k) := [\upsilon_1^t(k),..\upsilon_{N_k}^t(k)]^t$. The following recursive formula is derived in [3]:

$$\begin{bmatrix} V_o(k+1) \\ \upsilon(k+1) \end{bmatrix} = \begin{bmatrix} \phi(k+1,k) & \lambda(k+1,k) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_o(k) \\ \upsilon(k) \end{bmatrix} + \begin{bmatrix} H(k+1) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \beta(k+1) \\ \upsilon(k+1) \end{bmatrix} \qquad (3.8)$$

where

$$\phi(k+1,k) := diag(L_{k,k+1}, L_{k,k+1})\phi(\eta^k(z_c))$$

$$\lambda(k+1,k) := diag(L_{k,k+1}, L_{k,k+1})[\Psi_1(z_c)I_{6\times 6}\ \Psi_2 I_{6\times 6} \dots \Psi_{N_k} I_{6\times 6}]$$

The rotation matrix is

$$L_{k,k-1} = L(\sigma_k)L(\xi(z_c)) \qquad (3.9)$$

defines the relative angular orientation of $\mathcal{F}^k$ relative to $\mathcal{F}^{k-1}$.

Finally, let us define the *nodal spatial velocity* vector V(k) and the *nodal quasi-velocity vector* p(k) for the k^{th} body

$$V(k) := \begin{bmatrix} V_o(k) \\ \upsilon(k) \end{bmatrix}, p(k) := \begin{bmatrix} \beta(k) \\ \upsilon(k) \end{bmatrix} \qquad (3.10)$$

which allows us to write the recursion (3.8) in the form.

$$V(k) = \phi(k,k-1)V(k-1) + H(k)p(k), \quad k = 1, \dots, K, \quad V(0) = 0 \quad (3.11)$$

Our goal is to construct the spatial velocity vector and the kinetic energy function for the entire chain. Let us define the chain spatial velocity and quasi-velocity

$$V := [V^t(1),..,V^t(K)]^t, \quad p := [p^t(1),..,p^t(K)]^t \qquad (3.12)$$

so that we can write

$$V = \Phi H p, \qquad (3.13)$$

where

$$\Phi := \begin{bmatrix} I & 0 & \dots & 0 \\ \phi(2,1) & I & \dots & 0 \\ : & : & : & : \\ \phi(K,1) & \phi(K,2) & \dots & I \end{bmatrix}, \quad H := \begin{bmatrix} H(1) & 0 & \dots & 0 \\ 0 & H(2) & \dots & 0 \\ : & : & : & : \\ 0 & 0 & \dots & H(K) \end{bmatrix}$$

$$\phi(i,j) := \phi(i,i-1)\dots\phi(j+1,j), \quad i=2,..,K \text{ and } j=1,..,K-1$$

We assume that the kinetic energy for each link has been constructed (via finite element reduction) in the form

$$K.E._{link\ k} = \frac{1}{2} V^t(k) M_0(k) V(k) \qquad (3.14)$$

Then the kinetic energy function for the chain consisting of links 1 through K is

$$K.E._{chain} = \mathcal{T}(p,q) = \frac{1}{2} p^t \mathcal{M} p \qquad (3.15)$$

where the *chain inertia matrix* is

$$\mathcal{M} := H^*\Phi^* M \Phi H, \quad M := diag(M_o(1),..,M_o(K)) \qquad (3.16)$$

Remarks:

(1) Sliding joints: A sliding joint is one which admits one degree of freedom relative translation along a path defined in one of the bodies. They are easily accommodated for either rigid or deformable bodies within the framework described in [3].

(2) Finite element reduction: One approach to finite element reduction is based on collocation by splines. Our implementation of this method is described in [2]. It is simple and convenient for the class of models of interest herein.

(3) Poincaré's Equations: The above definitions and constructions provide the kinetic energy function in the form $\mathcal{T}(q,p) = p^t\mathcal{M}(q)p$. Hence, we reduce (2.9b) to the form:

$$\mathcal{M}(q)\dot{p} + \mathcal{C}(q,p)p + \mathcal{F}(q) = Q_p \qquad (3.17)$$

where

$$\mathcal{C}(q,p) := -\left[\frac{\partial(mp)}{\partial q} V\right]p + \frac{1}{2}\left[\frac{\partial(mp)}{\partial q} V\right]^t p + \sum_{j=1}^{m} p_j X_j^t U^t m$$

$$\mathcal{F}(q) := V^t(q)\frac{\partial \mathcal{V}(q)}{\partial q^t}, \quad Q_p := V^t(q)Q$$

Notice that Q_p denotes the generalized forces represented in the p-coordinate frame whereas Q denotes the generalized forces in the $\dot{q}$-coordinate frame (aligned with q). Q_p is actually more convenient because the quasi-velocities are usually represented in appropriate body frames.

(4) Taylor Linearization: If Q_p is constant, it makes sense to discuss equilibria of the system defined by (2.9a) and (3.17). An equilibrium point is defined as a value of the state (q,p) such that $\dot{q}$=0 and $\dot{p}$=0. From (2.9a) and the invertability of V(q) we find that p=0 at an equilibrium point. An equilibrium value of q then satisfies $f(q)=Q_p$. For convenience, let the equilibrium point of interest correspond to and q=0. A straight forward computation shows that the Taylor linearized dynamics are

$$\dot{q} = V(0)p \qquad (3.18a)$$

$$\mathcal{M}(0)\dot{p} + \mathcal{C}(0,0)p + \frac{\partial \mathcal{F}}{\partial q}(0)q = \Delta Q_p \qquad (3.18b)$$

4 Nonlinear Attitude Control via PFL

The approach to attitude control design considered herein derives from a now well established theoretical basis for control design by *feedback linearization* [6]. In recent work, including [2,3], we have tailored this technique to take advantage of the special structure of Lagrangian dynamics either in the form of classical Lagrange's equations or Poincaré's equations.

4.1 Partial Feedback Linearizing Control

The spacecraft models formulated above are of the from:

$$\dot{q} = V(q)p \qquad (4.1a)$$
$$\mathcal{M}(q,t)\dot{p} + \mathcal{C}(q,p,t)p + \mathcal{F}(q,t) = G\tau \qquad (4.1b)$$

The class of attitude control problems we invesigate is best characterized by partitioning the coordinate vector, and correspondingly the quasi-velocity vector, into two parts

$$q = \begin{bmatrix} \xi \\ u \end{bmatrix}, p = \begin{bmatrix} \omega \\ v \end{bmatrix} \qquad (4.2)$$

where ξ represents the controlled body attitude parameters and ω the corresponding body angular velocity, whereas u,v represent the remaining coordinates and velocities, respectively. Then in partitioned form, the equations are:

$$\dot{\xi} = \Pi(\xi)\omega \qquad (4.3a)$$
$$\dot{u} = \Sigma(\xi,u)v \qquad (4.3b)$$
$$M_\omega\dot{\omega} + N\dot{v} + F_\omega = G_\omega\tau \qquad (4.3c)$$
$$N^T\dot{\omega} + M_v\dot{v} + F_v = G_v\tau \qquad (4.3d)$$

Our goal is to regulate the outputs $y = \xi$. The concept of partial feedback linearization (PFL) is a general approach to the design of nonlinear control systems for a general class of systems with smooth nonlinearities [6]. Attitude control of spacecraft using feedback linearization was first used by Dwyer [5]. A PFL compensation for the system (4.3) is a nonlinear feedback law of the form

$$\tau = \mathcal{A}(\xi,\omega,u,v,t) + \mathcal{B}(\xi,\omega,u,v,t)\alpha \qquad (4.4)$$

which provides a closed loop attitude response in the linear, decoupled form

$$\ddot{\xi} = \alpha \qquad (4.5)$$

Specific conditions for the existence and construction of such controllers are given in Isidori [6]. Herein we describe the construction of PFL controllers for spacecraft modeled by Poincaré's equations.

The main constructive result is summarized in the following proposition:

Proposition 4.1: The PFL control for regulation of the outputs $y = \xi$ for the system defined by (4.3) takes the form of (4.4) with

$$\mathcal{A}=[G_{\omega}-NM_v^{-1}G_v]^{-1}\{F_{\omega}-NM_v^{-1}F_v+[NM_v^{-1}N^T-M_{\omega}]\Gamma^{-1}\frac{\partial\Gamma\omega}{\partial\xi}\Gamma\omega\} \qquad (4.6)$$

$$\mathcal{B}=[G_{\omega}-NM_v^{-1}G_v]^{-1}[M_{\omega}-NM_v^{-1}N^T]\Gamma^{-1}$$

proof: We prove the proposition by direct construction, in two steps. First, we use linearizing feedback to reduce (4.3c) to the form $\dot{\omega}=\beta$ which we then reduce to (4.5) by a second linearizing feedback. The composition of these to control laws gives the desired result. Equation (4.3d) can be solved for $\dot{v}$:

$$\dot{v}=-M_v^{-1}N^T\dot{\omega}-M_v^{-1}F_v+M_v^{-1}G_v\tau$$

which allows it elimination from (4.3c):

$$[M_{\omega}-NM_v^{-1}N^T]\dot{\omega}+F_{\omega}-NM_v^{-1}F_v=[G_{\omega}-NM_v^{-1}G_v]\tau$$

Now we choose the feedback control law:

$$\tau=[G_{\omega}-NM_v^{-1}G_v]^{-1}\left\{F_{\omega}-NM_v^{-1}F_v+[M_{\omega}-NM_v^{-1}N^T]\beta\right\}$$

which yields:

$$\dot{\omega}=\beta$$

Now, differentiation of (4.3a) provides:

$$\ddot{\xi}=\frac{\partial\Gamma\omega}{\partial\xi}\dot{\xi}+\Gamma(\xi)\dot{\omega}=\frac{\partial\Gamma\omega}{\partial\xi}\Gamma(\xi)\omega+\Gamma(\xi)\beta$$

Choose, the control law:

$$\beta=\Gamma^{-1}(\xi)\left\{\alpha-\frac{\partial\Gamma\omega}{\partial\xi}\Gamma(\xi)\omega\right\}$$

to obtain:

$$\ddot{\xi}=\alpha$$

and the desired composite linearizing control law is:

$$\tau=[G_{\omega}-NM_v^{-1}G_v]^{-1}\left\{F_{\omega}-NM_v^{-1}F_v+[M_{\omega}-NM_v^{-1}N^T]\Gamma^{-1}\left[-\frac{\partial\Gamma\omega}{\partial\xi}\Gamma\omega\right]\right\}$$

which is the stated result. §

Remarks:

(1) The linearizing control law is local if the parameterization of the angular configuration is local. However, there is some flexibility here because one may choose alternate parameterizations (e.g. Gibbs or Euler parameters), as appropriate to the problem. In either case, Γ has known singular points which limit the range of linearizability.

(2) In the specific problem of interest herein we have $G_{\omega}=I_3$ and $G_v=0$, so that (4.6) simplifies somewhat to:

$$\mathcal{A}=\{F_{\omega}-NM_v^{-1}F_v+[NM_v^{-1}N^T-M_{\omega}]\Gamma^{-1}\frac{\partial\Gamma\omega}{\partial\xi}\Gamma\omega\} \qquad (4.7a)$$

$$\mathcal{B}=[M_{\omega}-NM_v^{-1}N^T]\Gamma^{-1} \qquad (4.7b)$$

(3) The invertibility of M_v is assured because it is an inertia matrix for a physical subsystem which is consequently a positive definite matrix.

(4) Equation (4.5) may be rewritten

$$\dot{z}=Az+B\alpha,\ A:=\begin{bmatrix}0&I_3\\0&0\end{bmatrix},B:=\begin{bmatrix}0\\I_3\end{bmatrix} \qquad (4.8a)$$

we may easily choose a stabilizing control for (4.5)

$$\alpha=K_p\xi+K_r\dot{\xi}=Kz \qquad (4.8b)$$

4.2 Adaptive PFL Control

Because feedback linearization is a model based approach to control system design, it is necessary to anticipate some sensitivity to model uncertainty. In the present case, it is reasonable to assume that the kinematics are precisely known but that the dynamics are not. Thus, we consider the situation where the model contains uncertain parameters, denoted ϑ, which belong to a bounded set $\mathfrak{J}$. Equations (4.3), may be rewritten with these parameters explicitly shown

$$M_{\omega}(\vartheta)\dot{\omega}+N(\vartheta)\dot{v}+F(\vartheta)_{\omega}=G_{\omega}\tau \qquad (4.9a)$$

$$N(\vartheta)^T\dot{\omega}+M_v(\vartheta)\dot{v}+F_v(\vartheta)=G_v\tau \qquad (4.9b)$$

Because of its physical meaning, the invertibility of $M_v(\vartheta)$ is preserved for all values of $\vartheta\in\mathfrak{J}$. Consequently, a feedback linearizing control exists for all parameter values. Indeed, the control (4.4) as constructed via Proposition 4.1 is a parameter dependent control, which we rewrite in the form

$$\tau(\vartheta)=\mathcal{A}_{\vartheta}(\xi,\omega,u,v,t)+\mathcal{B}_{\vartheta}(\xi,\omega,u,v,t)\alpha \qquad (4.10)$$

The idea is to implement (4.9) with ϑ replaced by an estimate $\hat{\vartheta}$. When the estimated control $\tau(\hat{\vartheta})$ is applied, the system is not exactly feedback linearized and a simple computation shows that (4.5) is replaced by

$$\ddot{\xi}=\alpha+\Delta(\hat{\vartheta},\vartheta,\xi,\omega,u,v,t) \qquad (4.11)$$

The following proposition provides a parameter adaptive feedback linearizing control law.

Proposition 4.2: Consider the system defined by (4.3a&b) and (4.9) with control $\tau(\hat{\vartheta})$ where $\tau(\cdot)$ is given by (4.10) and α by (4.8b). Suppose that the residual Δ defined in (4.11) has the form

$$\Delta(\hat{\vartheta},\vartheta,\xi,\omega,u,v,t)=\Psi(\xi,\omega,u,v,t)(\vartheta-\hat{\vartheta}) \qquad (4.12)$$

Then an asymptotically stable controller is achieved with the parameter estimator

$$\dot{\hat{\vartheta}}=Q\Psi^t(\xi,\omega,u,v,t)B^tPz \qquad (4.13)$$

where P is a symmetric, positive definite solution of

$$(A+BK)^tP+P(A+BK)=-I \qquad (4.14)$$

and Q is any symmetric, positive definite marix.

proof: Various forms of this result are well known, e.g. [13].

5 Summary of Simulation Results

In the following paragraphs we describe simulation results which compare linear and nonlinear (PFL) controllers for attitude control of a prototype space station. Prior to consideration of flexible platform studies were conducted with a rigid platform.

5.1 System Configuration

The space station with MRMS is idealized to be composed of four articulated elements: the space station main body (body 1), the MRMS base (body 2), the upper (inner) MRMS arm (body 3), and the lower (outer) MRMS arm (body 4). It is asumed that the MRMS base, body 2, can move along a fixed path on the space station, body 1, while body 3 is joined to body 2 and body 4 to body 3 via joints with up to three rotational degrees of freedom. The setup is illustrated in figure 2. We consider the case where the MRMS joints are each resticted to one degree of freedom: joint 3 admits only rotations about the z-axis in the f^3 frame and joint 4 about the x-axis in the f^4 frame.

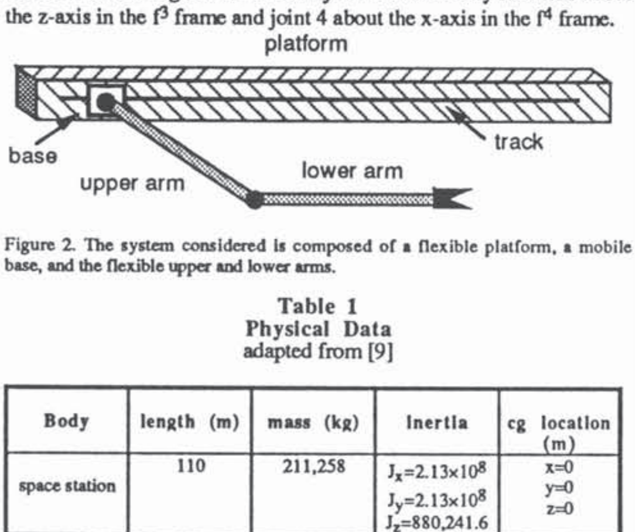

Figure 2. The system considered is composed of a flexible platform, a mobile base, and the flexible upper and lower arms.

Table 1
Physical Data
adapted from [9]

Body	length (m)	mass (kg)	Inertia	cg location (m)
space station	110	211,258	$J_x=2.13\times10^8$ $J_y=2.13\times10^8$ $J_z=880,241.6$	x=0 y=0 z=0
mobile base	1.5	316.9	$J_x=178.25$ $J_y=178.25$ $J_z=356.5$	x=0 y=0 z=0
upper arm	14.3	3169	$J_x=54,002$ $J_y=54,002$ $J_z=0$	x=0 y=0 z=7.15
lower arm	14.3	3169	$J_x=54,002$ $J_y=54,002$ $J_z=0$	x=0 y=0 z=7.15

The platform is treated as a flexible beam for which a model is developed in accordance with the finite element method described in [5], using collocation by splines as applied to a Timoshenko formulation of beam dynamics. Even with only two elements, the resultant system is excessively stiff. Thus, we reduce the system to retain 4 flexure degrees of freedom (8 modes) by retaining the so-called long wavelength dynamics, so that angular deformation coordinates are eliminated. Of these, 4 modes are near the control bandwidth (natural frequencies of about 3 rad/s) and the others are outside the bandwidth (approximately 10 rad/s). The result is 13 degrees of freedom with configuration variables:

- $R \in \mathbb{R}^3$, the location of point O_1 on body 1 relative to inertial space.
- $L_1 \in SO(3)$, the relative angular orientation of $\mathcal{F}^1$ with respect to inertial space.
- $\tilde{\eta}_i \in \mathbb{R}^2$, i=1,..,N(=2) platform deformation coordinates
- $\zeta \in \mathbb{R}$, the location of the MRMS base along undeformed track in the frame $\mathcal{F}^1$.
- $\psi_{32} \in \mathbb{R}$, the relative angular orientation of $\mathcal{F}^3$ with respect to $\mathcal{F}^2$.
- $\phi_{43} \in \mathbb{R}$, the relative angular orientation of $\mathcal{F}^4$ with respect to $\mathcal{F}^3$.

The joint quasi-velocities are $\beta(1)=(\omega_1,v_1)$ the linear velocity v_1 and the angular velocity ω_1 of f^1, the linear velocity $\beta(2)=v_{2z}$ for joint 2, and the relative angular velocities $\beta(3)=\omega_{32}$ and $\beta(4)=\omega_{43}$ for joints 3 and 4.

Table 1 provides the physical data used in the simulation studies. We assume that the beam is a uniform, square boxbeam with outside dimension of 5 m and has the following material properties

desity $\rho = 7.860\times10^3$ kg/m^3
modulus of elasticity $E = 200\times10^8$ N/m^2
shear modulus $G = 79\times10^8$ N/m^2

All of the platform characteristics except dissipation properties follows from these assumptions. A material dissipation model of the type described in [3] is assumed. In addition, we assume some form of active or passive vibration suppression provides additional damping. Even so, the dominant modes of the structure are very lightly damped as will be seen in the simulation results.

5.2 System Equations

The dynamical equations of motion for the composite system including the space station with MRMS have been derived in terms of Poincaré's equations. In this study we prescribe the MRMS motion and determine the corresponding SSF response. The MRMS motion is defined by prescribing the MRMS acceleration and computing the resultant motion using the kinematic constraints. Thus, we have

$$\begin{bmatrix} \dot{\zeta} \\ \dot{\psi}_{32} \\ \dot{\phi}_{43} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{2z} \\ \omega_{32z} \\ \omega_{43x} \end{bmatrix} \quad (5.1a)$$

$$\begin{bmatrix} \dot{v}_{2z} \\ \dot{\omega}_{32z} \\ \dot{\omega}_{43x} \end{bmatrix} = \begin{bmatrix} a_{2z} \\ a_{32z} \\ a_{43x} \end{bmatrix} \quad (5.1b)$$

In all of the subsequent simulations we use the above MRMS motion model with the accelerations a_{2z}, a_{32z}, a_{43x} prescribed as constants. There remains a great deal of flexibility in this model because in addition to specifying the accelerations the initial conditions on velocities and configuration variables may also be prescribed.

With the motion of the MRMS prescribed, the equations governing the response of the space station are obtained by stripping off the first ten of Poicaré's equations for the composite system.

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{R} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} \Gamma(\xi_1) & 0 & 0 \\ 0 & L_1(\xi_1) & 0 \\ 0 & 0 & I_{4\times4} \end{bmatrix} \begin{bmatrix} \omega_1 \\ v_1 \\ \upsilon \end{bmatrix} \quad (5.2a)$$

$$\mathcal{M}_{11} \begin{bmatrix} \dot{\omega}_1 \\ \dot{v}_1 \\ \dot{\upsilon} \end{bmatrix} = \begin{bmatrix} \tilde{\omega}_1 & \tilde{v}_1 & 0 \\ 0 & \tilde{\omega}_1 & 0 \\ 0 & 0 & 0_{4\times4} \end{bmatrix} \mathcal{M}_{11} \begin{bmatrix} \omega_1 \\ v_1 \\ \upsilon \end{bmatrix} + \mathcal{G} \begin{bmatrix} a_{2z} \\ a_{32z} \\ a_{43x} \end{bmatrix} + \begin{bmatrix} \tau \\ 0 \end{bmatrix} \quad (5.2b)$$

$$\mathcal{G} := \left\{ -\left[\frac{\partial(\mathcal{M}_1)}{\partial \zeta} p \ \frac{\partial(\mathcal{M}_1)}{\partial \psi_{32}} p \ \frac{\partial(\mathcal{M}_1)}{\partial \phi_{43}} p \right] - \mathcal{M}_{12} + \begin{bmatrix} \tilde{\omega}_1 & \tilde{v}_1 & 0 \\ 0 & \tilde{\omega}_1 & 0 \\ 0 & 0 & 0_{4\times4} \end{bmatrix} \mathcal{M}_{12} \right\}$$

5.3 Simulation Results

Simulation studies were conducted for both rigid and flexible platform models using linear, PFL and adaptive PFL control laws. The linear controls were obtained by applying the PFL construction to the linearized model, i.e., they are standard linear decoupling controls. Thus, allowing meaningful comparison of the linear and PFL controls.

Linear Control: In the rigid platform case the linear controller produced acceptable attitude error responses in the absence of Manuoulator motion and with initial attitude (Euler angle) offsets of .5 radians on each axis. However, large motions of the manipulator caused instability, just as obseved in [14].

The flexible platform case was quite different. Table 2 lists the open and closed loop eigenvalues for different feedback gain values. The open loop set consists of 12 zero eigenvalues corresponding to the rigid body dynamics and an additional 8 corresponding to the platform flexure dynamics. The second column lists the eigenvalues resulting from a design intended to achieve the same attitude response as had been achieved in a study of the rigid body case. Notice that the first 14 eigenvalues correspond to the "zero dynamics" and remain fixed as the attitude gain is "detuned" in column three. The zero dynamics modes include the 3 rigid body translation modes and 4 cantilevered beam modes of the platform. Although the nominal closed loop linear system is stable, application of the linear regulator to the nonlinear simulation produced divergent trajectories with initial attitude errors as small as .01 radians. This is due to destabilizing inertial crosscoupling between the flexible and rigid body dynamics. Detuning of the closed loop appeared appropriate in order to reduce slewing rates and hence platform flexure. The detuned regulator of column three did produce convergent trajectories with initial errors of .01 radians, but this appeared to be close to the limit of the domain of attraction.

Table 2
Open and Closed Loop Eigenvalues

Open Loop	Nominal Closed Loop (k)	Detuned Closed Loop (k/8)
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	-0.1763 + 3.3205i	-0.1763 + 3.3205i
0	-0.1763 - 3.3205i	-0.1763 - 3.3205i
0	-0.1763 + 3.3205i	-0.1763 + 3.3205i
0	-0.1763 - 3.3205i	-0.1763 - 3.3205i
0	-0.1364 + 1.6505i	-0.1364 + 1.6505i
0	-0.1364 - 1.6505i	-0.1364 - 1.6505i
-10.4212 +10.5963i	-0.1364 + 1.6505i	-0.1364 + 1.6505i
-10.4212 -10.5963i	-0.1364 - 1.6505i	-0.1364 - 1.6505i
-10.8762 +10.5876i	-0.2000 + 0.2040i	-0.0250 + 0.0979i
-10.8762 -10.5876i	-0.2000 - 0.2040i	-0.0250 - 0.0979i
-0.2053 + 3.3267i	-0.2000 + 0.2040i	-0.0250 + 0.0979i
-0.2053 - 3.3267i	-0.2000 - 0.2040i	-0.0250 - 0.0979i
-0.2053 + 3.3290i	-0.2000 + 0.2040i	-0.0250 + 0.0979i
-0.2053 - 3.3290i	-0.2000 - 0.2040i	-0.0250 - 0.0979i

The significance of the nonlinear interactions which arise through the inertial couplings is quite striking. It is anticipated that further detuning would lead to a larger domain of attraction for the stable equilibrium point, although we have not confirmed this. Even so, it is clear that the achievable performance with linear regulators is severely limited. Performance specifications for MRMS motion and attitude regulation will not ,in practice, approach the levels demanded herein. For example, we impose an MRMS translation of 18m in 60 sec, whereas, Wie et al [14] impose a translation of 5m in 300 sec.

PFL Control: We first consider attitude regulation with an MRMS maneuver combined with initial attitude errors and with perfect knowledge of all parameters. The excellent PFL control results are illustrated in Figures 3. The system easily accommodates .5 radians attitude error on each axis with a simultaneous MRMS motion.

The effect of a minimal 5% stiffness uncertainty on the PFL controller, however, results in seriously degraded, unstable regulation. This sensitivity is consistent with our prior observations about the linear regulator and, again, it is likely that sensitivity would be substantially reduced by detuning of the stabilizer and reduction of the rate of MRMS motion. Figure 4 illustrates the adaptive PFL with MRMS motion and 5% stiffness uncertainty. Adaptation almost restores ideal performance. Similar results have been achieved with 10% uncertainty. However, 15% uncertainty results in unacceptable performance.

6 Conclusions

This paper summarizes results of a study of the application of partial feedback linearization methods to the attitude control of an articulated spacecraft representative of the Space Station Freedom with a Mobile Remote Manipulator System. Computer studies contrast linear state feedback regulators with PFL type attitude stabilizers. The results confirm previous observations that MRMS motion can significantly degrade and even destabilize attitude regulation when linear controllers are applied to this highly nonlinear dynamical system. Our results show that in the flexible case the linear regulator must be significantly detuned in order to achieve stable responses. As a matter of fact, even with detuning, the attitude errors must be very small in order to observe the behavior predicted by linear theory. Parameter uncertainty is not tolerable. Although the studies conducted to date are far from exhausting, it is clear that PFL design is promising. It is shown that the PFL controller performs quite well with perfect knowledge (no parameter uncertainty) both with respect to decoupling and stabilization. However, performance deteriorates rapidly with even small parametric uncertainties. Adaptive PFL is shown to restore the excellent PFL performance with stiffness uncertainties of up to 10%. Controller detuning will certainly improve robustness and studies which address the tradeoff between performance and sensitivity would be required in any given design situation.

References

1. V. I. Arnold, V. V. Kozlov and A. I. Neishtadt, "Mathematical Aspects of Classical and Celestial Mechanics," in *Dynamical Systems III*, V. I. Arnold, ed., Springer-Verlag Berlin, 1988.
2. W. H. Bennett, H. G. Kwatny, C. LaVigna and G. L. Blankenship, "Nonlinear and Adaptive Control of Flexible Space Structure," in *Control of Systems With Inexact Dynamic Models*, DSC-Vol. 33, ASME 1991, pp. 73-91.
3. W. H. Bennett, H. G. Kwatny, and M.-J. Baek, "Nonlinear Dynamics & Control of Flexible, Articulated Spacecraft: Application to SSF/MRMS," AIAA Conf. on GNC, August 1992.
4. N. G. Chetaev, *Theoretical Mechanics*, Springer-Verlag, New York, 1989.
5. T. A. W. Dwyer, "Exact Nonlinear Control of Large Angle Rotational Maneuvers," *IEEE Trans. on Auto. Contr.*, Vol. AC-29, No. 9, pp.769-774, 1984.
6. A. Isidori, *Nonlinear Control Systems: An Introduction*, 2nd ed., Springer-Verlag: New York, 1989.
7. A. Jain, "Unified Formulation of Dynamics for Serial Rigid Multibody Systems," *AIAA Jrl. Guidance and Control*, Vol. 14, No.3, pp. 531-542, 1991.
8. H. G. Kwatny, F. M. Massimo and L. Y. Bahar, "The Generalized Lagrange Formulation for Nonlinear RLC Networks", *IEEE Trans. on Circuits and Systems*, Vol. CAS-29, No. 4, pp. 220-233, April, 1982.
9. H. W. Mah, Modi, V. J., Morita, Y. and Yokoto, H., "Dynamics and Control During Slewing Maneuvers," *Acta Automatica*, Vol. 19, No. 2, pp.125-143, 1989.
10. L. Meirovich, *Analytical Methods in Vibrations*, McMillan: New York, 1967.
11. Ju. I. Neimark and Fufaev, N. A., *Dynamics of Nonholonomic Systems*, American Mathematical Society, 1972.
12. G. Rodriguez, "Kalman Filtering, Smoothing and Recursive Robot Arm Forward and Inverse Dynamics," *IEEE Jrl. of Robotics and Automation*, Vol. RA-3, No. 6, pp. 624-639, 1987.
13. S. S. Sastry and A. Isidori, "Adaptive Control of Linearizable Systems," *IEEE Trans. Auto. Contr.*, Vol. 34, No. 11, pp. 1123-1131, 1989.
14. B. Wie, A. Hunt and R. Singh, "Multibody Interaction Effects on Space Station Attitude Control and Momentum Management," *AIAA J. Guidance, Navigationa nd Control*, Vol. 13, No. 6, Nov.-Dec. 1990.

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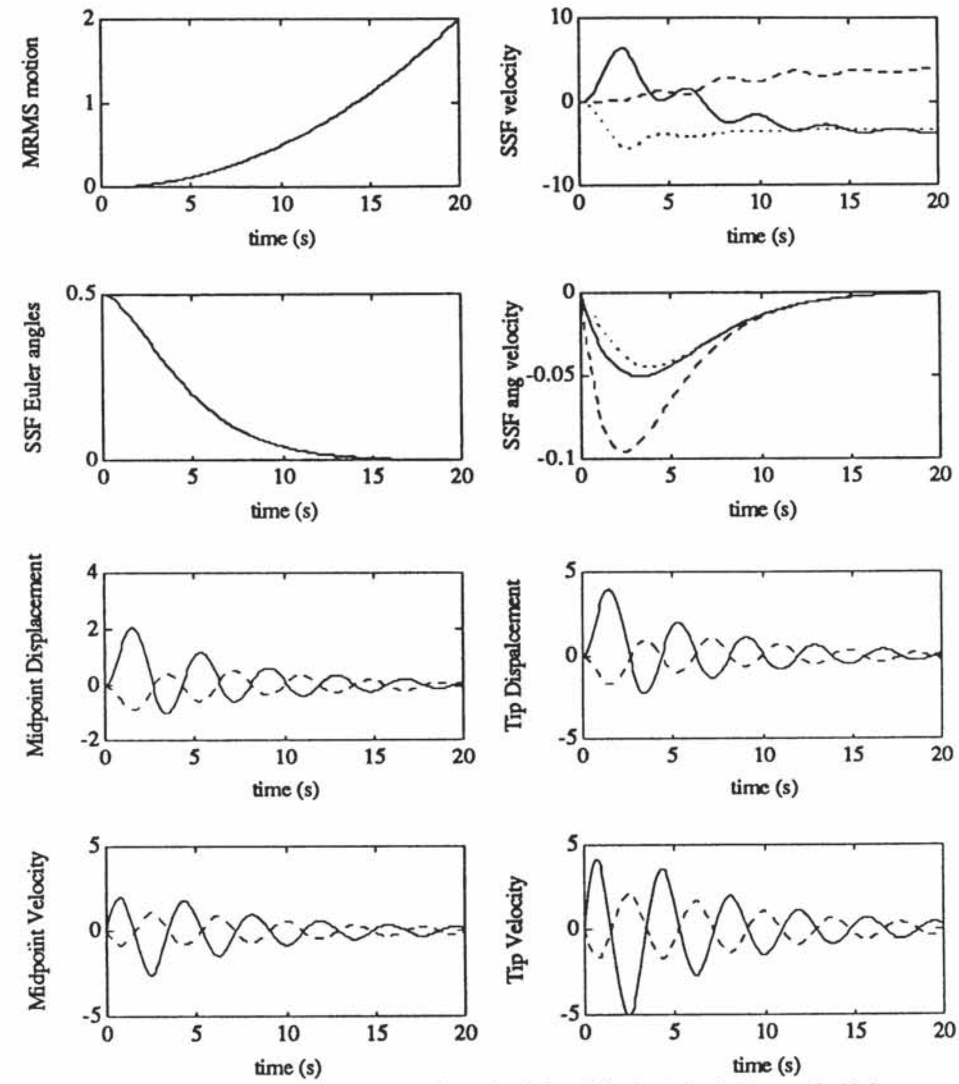

Figure 3. The effectiveness of combined PFL decoupling and attitude stabilization is clearly illustrated in this figure.

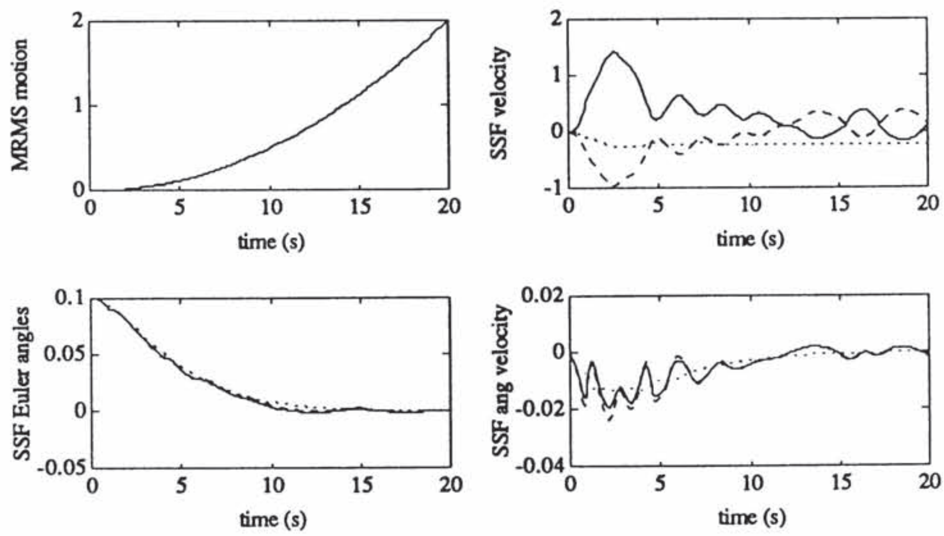

Figure 4. Adaptive PFL with 5% uncertainty.